Nuclear Theory - Course 127

FLUX DISTRIBUTION AND CRITICAL SIZE

Discussions so far have centred around the reduction of neutron losses, due to radiative capture, so as to increase some, or all, of the factors in the four factor formula and, thus, maintain a chain reaction. Neutrons are also lost through escape or leakage out of the reactor and consideration will now be given to neutron conservation by reduction of these losses. Neutron leakage out of the reactor may be reduced in one of two ways:

(a) By increasing the size of the reactor.

(b) By using a reflector.

The first of these two methods will now be discussed.

The multiplication factor, k_∞ , for a reactor of infinite size is connected with η , ϵ , p and f by the four factor formula:

$$k_\infty = \eta \epsilon p f \quad \text{.......................................(1)}$$

If there is no leakage of neutrons out of the system, the condition for criticality is

$$k_\infty = 1$$

When neutron leakage has to be considered, the multiplication factor, k_e, (known as k - effective), allows for this leakage as follows:

$$k_e = k_\infty - \text{neutron leakage} \quad \text{........................(2)}$$

It may be shown that:

$$k_e = k_\infty - M^2B^2 \quad \text{..................................(3)}$$

where M is the neutron migration length, which depends on the core composition, and B^2 is the buckling, which depends on the size and shape of the reactor and on the flux distribution. For instance in a cylindrical reactor:

$$B^2 = \frac{(2.405)^2}{R_e^2} + \frac{\pi^2}{L_e^2} \quad \text{..........................(4)}$$

where R_e is known as the EXTRAPOLATED radius and L_e the extrapolated length. R_e and L_e are slightly greater than the physical radius and length respectively. These quantities will be more clearly defined later in the lesson.

From the expression for B^2, (equation (4)), it is clear that the value of B^2 and, therefore, the neutron leakage will decrease if R or L increase.

Critical Size of a Reactor

It was stated above that neutron leakage from a reactor can be decreased by increasing the dimensions of the reactor. This is also evident from simple geometric considerations. As the volume of the reactor increases, the rate of fissioning increases. However, the surface area also increases and so, therefore, does the neutron leakage. When the dimensions of a reactor are increased, the volume increases faster than the surface area. Hence, the number of neutrons produced in the reactor increases faster than the neutron leakage and there is a net increase in the production of neutrons.

When the size of the reactor is such that the number of neutrons produced is exactly the same as the neutrons removed by fission, radiative capture and leakage, the chain reaction is maintained. The reactor is then critical, (with $k_e = 1$), and the size of the reactor is, then, its CRITICAL SIZE. The critical size will, therefore, be reached when neutron leakage has been reduced sufficiently for $k_e = 1$.

The critical size referred to is the minimum critical size with new fuel. Any size less than the minimum critical size cannot go critical. However, a reactor of this size would not remain critical since the quantity of fuel is being reduced by fissions and fission products, which absorb neutrons, accumulate. Thus, the reactor size is usually substantially greater than its minimum critical size.

A spherical reactor will have the smallest critical size because a sphere will have the smallest surface area for a given volume. A cylindrical reactor will have a bigger critical size that a spherical one and a critical cubical reactor will be bigger still. A spherical reactor would be somewhat difficult to build and, therefore, reactor of cylindrical shapes are usually chosen.

Neutron Flux Distribution

The thermal neutron flux, ϕ, in a reactor, is the quantity that determines the number of fissions that take place per second. The higher the neutron flux, at any point in a reactor, the higher the rate of fissioning at that point and the greater the power produced at that point.

The neutron flux may be defined in a number of ways:

(a) If ϕ is the neutron flux at a point the number of neutron induced reactions per cc per second at that point will be given by:

$$\text{No. of reactions/cc/sec} = \phi N \sigma$$

where N is the number of reacting nuclei per cc at that point and σ the cross-section for that reaction.

If σ_f is the fission cross-section then $\phi N \sigma_f$ will be the number of fissions/cc/sec.

(b) The thermal neutron flux, at a point in a reactor, is the product of the thermal neutron density, n, at that point and the average speed of the thermal neutrons.

ie, $\phi = nv$

(c) The thermal neutron flux, at a point in a reactor, is the number of thermal neutrons per second, travelling in all possible directions, which cross a unit area placed at that point.

Definitions (a) and (b) are more mathematically correct and more acceptable than (c). However, definition (c) sometimes helps to give some physical significance to neutron flux. The unit used for neutron flux is usually neutrons/cm^2/sec.

The thermal neutron flux need not be the same at every point in a reactor. In fact, the flux is usually a maximum at the centre and has a distribution across a reactor which is characteristic of the shape of the reactor, provided that nothing is done to alter this natural distribution.

In a cylindrical reactor, shown in Fig. 1, there are two directions in which the flux distribution is considered. These directions are along the axis, Oz, and along the radial direction, Or from the centre of the reactor.

The axial flux distribution is given by:

$$\phi = \phi_m \cos\left\{\frac{\pi z}{L_e}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

That is to say, the flux at a distance z from O, along Oz in either direction, is given by the above equation, ϕ_m being the maximum flux at O.

The flux at any distance, r, from O is given by:

$$\phi = \phi_m J_o\left\{\frac{2.405r}{R_e}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

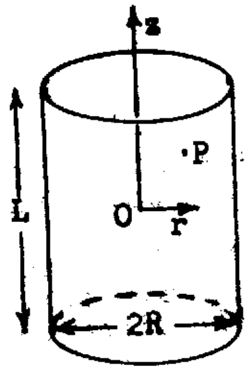

Fig. 1

where ϕ_m is again the maximum flux at O and J_o is a special function known as a zero order Bessel function. However, the radial flux distribution is given, very closely by a simpler cosine formula:

$$\phi = \phi_m \cos\left\{\frac{\pi r}{2R_e}\right\} \quad ....(7)$$

The quantities R_e and L_e are the extrapolated radius and length which were mentioned earlier in the lesson.

Fig. 2 shows both these cosine distributions graphically and also shows the significance of R_e and L_e.

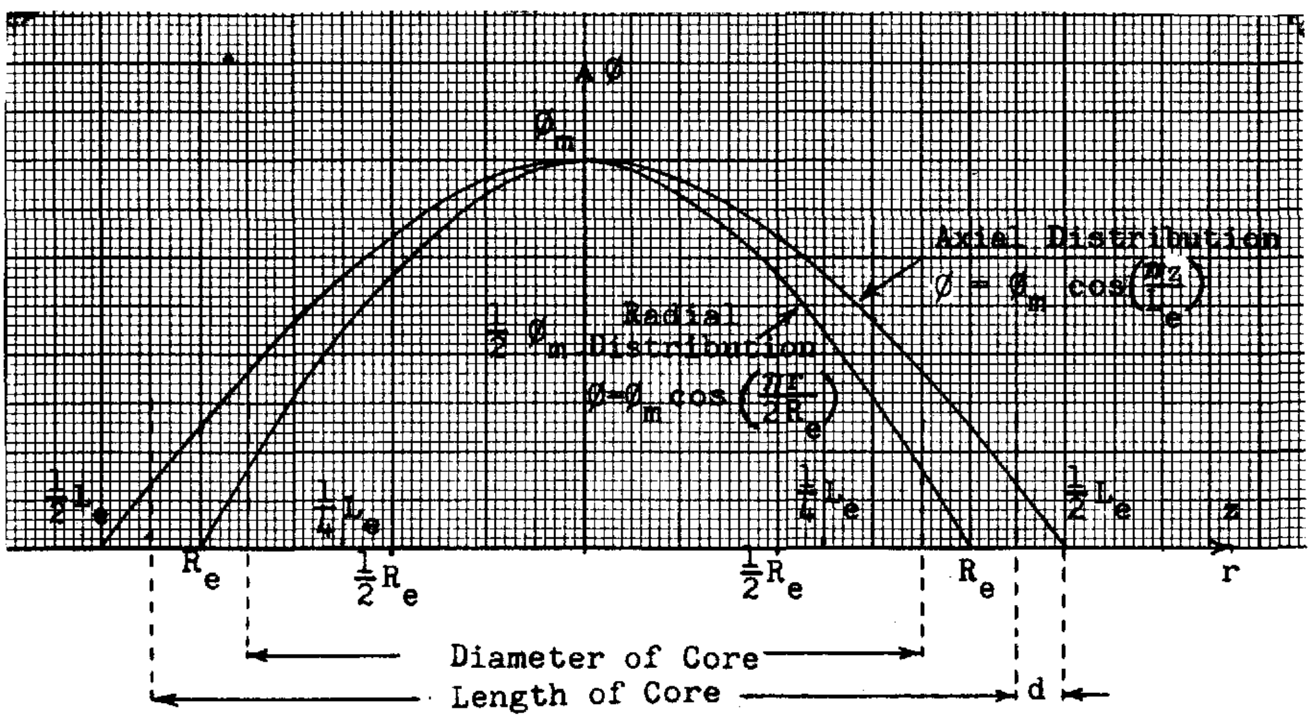

Fig. 2

It may be seen that the flux falls off to zero at some distance, d, outside the physical boundary of the core. This distance, d, is known as the EXTRAPOLATION DISTANCE. Thus if R and L are the actual physical dimensions of the core:

$$R_e = R + d$$

and $$L_e = L + 2d$$

The flux at any point P (Fig. 1), in the reactor, with axial coordinate z and radial coordinate r would be given by:

$$\phi = \phi_m \cos\left\{\frac{\pi r}{2R_e}\right\} \cos\left\{\frac{\pi z}{L_e}\right\} \quad \text{.......................(8)}$$

Effect of Neutron Flux Distribution on Reactor Power

An examination of the flux distribution graphs will show that:

(a) The thermal neutron flux has a definite maximum value at the centre but falls to nearly zero at the edge of the reactor. Since the rate of fissioning and, consequently, the power depends on $\phi\Sigma_f$, the maximum power is being produced at the centre and very little power is being produced from the fuel in the outer regions of the core.

(b) The total power produced by the reactor depends on the average thermal neutron flux, the relationship between the average flux, ϕ_a, and the power P (in Megawatts), being:

$$\phi_a = 3 \times 10^{12} \frac{P}{U} \quad \text{................................(9)}$$

where U is the total weight of uranium fuel in the reactor, in tonnes.

Thus, the power required and the average flux determine the total fuel loading and the core size. With the flux distributions, shown in the cylindrical case, ϕ_a is only 27.5% of the maximum flux, ϕ_m, ie, the maximum flux is 3.6 times greater than the average.

The average flux can only be increased, in a bare reactor, by increasing the maximum flux. However, the maximum flux is usually limited by the maximum fuel heat rating or by the severity of the Xenon transient. The maximum fuel rating will be reached much sooner in the centre of the reactor and so there is very poor utilization of the rest of the fuel.

<u>Example</u>

In NPD $\phi_m = 8 \times 10^{13}$ n/cm^2/sec

Hence $\phi_a = 2.2 \times 10^{13}$ n/cm^2/sec

Therefore $U = \dfrac{3 \times 10^{12} \times 80}{2.2 \times 10^{13}} = \underline{10.9 \text{ tonnes}}$

In Douglas Point the radial flux distribution is deliberately flattened over a portion of the reactor to increase ϕ_a.

ASSIGNMENT

1. (a) How is the effective multiplication factor connected with the neutron leakage?

   (b) On what two factors does the neutron leakage depend?

2. (a) Why does relative neutron leakage decrease as the reactor dimensions increase?

   (b) What is meant by the critical size of a reactor?

   (c) What reactor shape would have the smallest critical size and why is this so?

3. Define thermal neutron flux and specify the units in which it is measured.

4. (a) Write down the equations giving the approximate radial and axial flux distributions in a cylindrical reactor.

   (b) What two disadvantages result from this type of flux distribution?

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